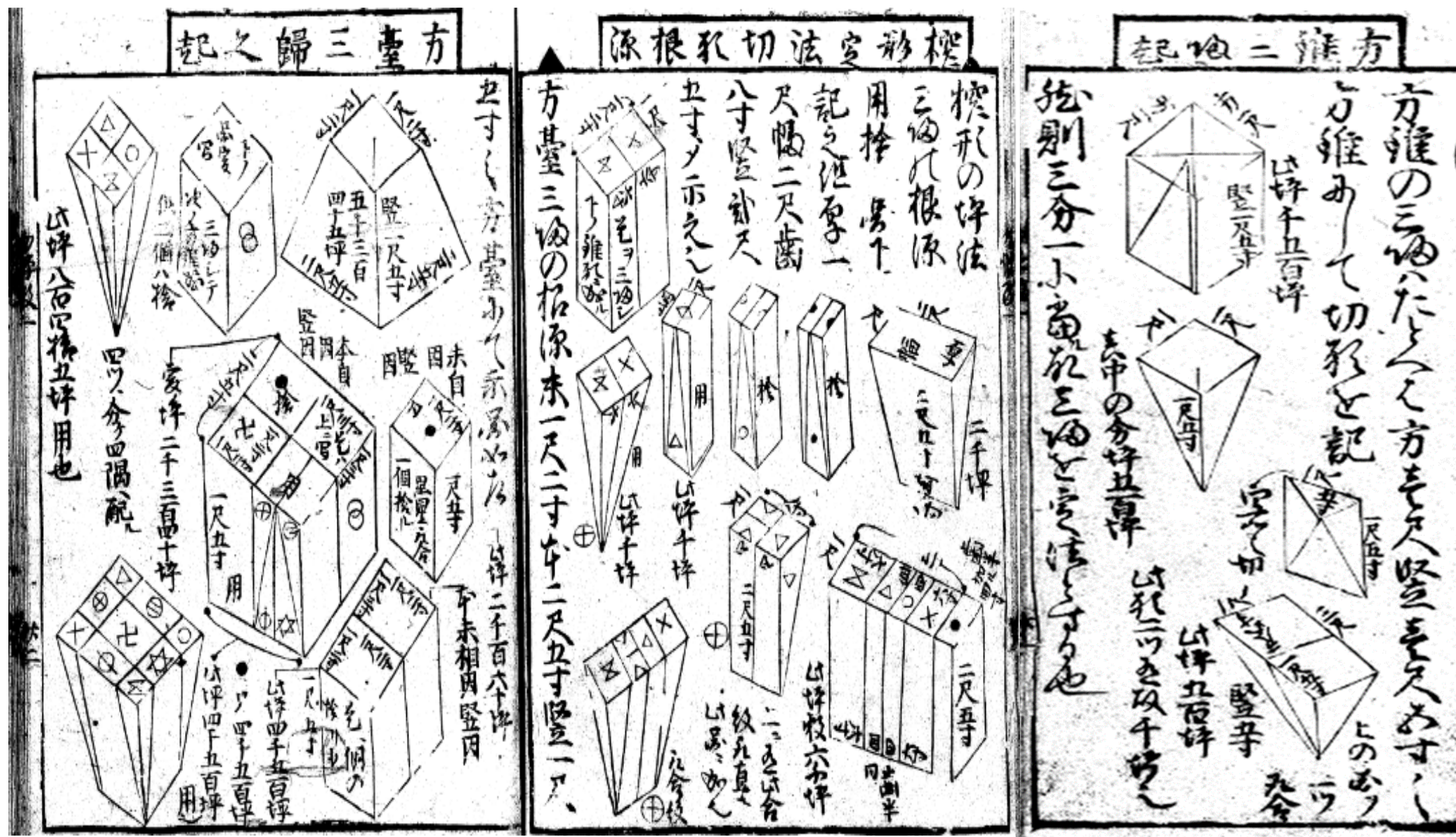


竜ヶ崎第一高等学校 白幡探究Ⅰ 数学領域

さまざまな立体の体積を求める公式

70th 1年B組 乙班

原文 Original



キーワード

・四角錐

・ $\frac{1}{3}$

Keyword

・QUADRANGULAR PYRAMID

・ONE THIRD

現代語訳 Modern Translation

正四角錐の体積を求める時に3分の1を掛ける理由は、たとえば底辺の正方形の1辺が1尺、高さが1尺5寸の正四角柱として、切り取った形を記す。
そうであるなら全体の3分の1になるので3分の1を掛けることを公式とする。

楔形の体積を求める時に3分の1を掛ける理由は下の図に記す。
底面のたて1尺、横2尺、歯の幅を8寸で全体の高さを2尺5寸としてここに示す。

正四角錐台の体積を求める時に3分の1を掛ける理由は、上底の正方形の1辺が1尺2寸、下底の正方形の1辺が2尺5寸、高さ1尺5寸の正四角錐台を使って左に図を示す。

係 國定 河合

英語訳 English Translation

There is a one a third regular quadrangular pyramid.
As a regular quadrangular prism which height is 1^{shaku 5sun}, each side of the base is 1^{shaku}, the cut off shape put down.
So dividing by three is the rule.

The way to solve the wedge figure

1 There is a selection figure whose the length of the base is 1^{shaku} long and 2^{shaku} wide, the length of the thin part is 8^{sun}.

2 The rule to divide regular pyramid base by three is truncated square pyramid. The surface of a square of a side is 1^{shaku 2sun}. The bottom of a square of a side is 2^{shaku 5sun}.

係 小滝 小田原

英語訳 English Translation

I I think the formula for the volume of the quadrangular pyramid by using the quadrangular prism.

I I use the quadrangular prism which Each side of the base is 1^{shaku} height is 1^{shaku 5sun}.

I cut the quadrangular pyramid which height is equal to solid of ①

Then I find that solid of ② volume is a one-third of the quadratic prism of ①.

From this, formula for the volume of the quadrangular pyramid becomes like the following.

The volume of the quadrangular pyramid = base area \times height $\times \frac{1}{3}$

II I think about the formula for the volume of the wedge shaped.

I I use the wedge shaped solid which the length of the tooth is 8^{sun}, width is 1^{shaku}, Thickness is 1^{shaku} and height is 2^{shaku 5sun}. In addition, the volume is 2000^{tsubo}.

I prepare quadratic prism that the length of the base is 1^{shaku}, the wide of the base is 2^{shaku 4sun} and height is as well as the solid of ①. This solid's volume is 6000^{tsubo}.

I distribute ② solid like a figure.

A The quadrangular prism that length is 1^{shaku}, width is 1^{shaku 2sun} and height is 2^{shaku 5sun}.

B Three quadratic prisms that length is 1^{shaku}, width is 4^{sun} and height is 2^{shaku 5sun}.

I cut and bring down a quadrangular pyramid that height is 2^{shaku 5sun} and the volume is a one-third of the ③ A solid from ③ A solid.

I use only one of three quadratic prisms of B. And transform it like figure.

It becomes wedge-shaped when I match the solid made of ④ ⑤.

That is to say, this solid is one-third of quadrangular prism of ②. Thus, this solid volume is 2000^{tsubo}.

The volume of ② is 1^{shaku 5sun} \times 2^{shaku 4sun} \times 2^{shaku 5sun}.

So wedge is 1^{shaku} \times 2^{shaku 4sun} \times 2^{shaku 5sun} $\times \frac{1}{3}$ = 1^{shaku} \times (2^{shaku} + 4^{sun}) \times 2^{shaku 5sun} $\times \frac{1}{3}$

Thus, formula of the volume of wedge is as follows.

The volume of the wedge = thickness \times (width + teeth $\times \frac{1}{2}$) \times height $\times \frac{1}{3}$

III I think about the formula for the volume of the square frustum that the base is square.

I I use the square frustum that each side of the lower base is 2^{syaku 5sun} each side of upper base is 1^{syaku 2sun} and height is 1^{syaku 5sun}.

Next, I prepare the quadrangular prism that each side of the base is 1^{syaku 5sun} and height is 1^{syaku 5sun} and I make a solid ① based on this solid.

A The quadrangular prism that each side of the base is 1^{syaku 2sun} and the height is 2^{syaku 5sun}.

B The quadrangular prism that the length of the base is 1^{syaku 2sun} the width of the base is 1^{syaku 3sun} and the height is 2^{syaku 5sun}.

C Next, I prepare the quadrangular prism that each side of the base is 1^{syaku 3sun} and the height is 2^{syaku 5sun}.

I cut the above mentioned solid

I put slits in solid of B a figure and divide it into four quadrangular prisms.

I make a quadrangular pyramid which the height is same, the volume is one-third of the solid of C.

I use the solid of A as it is. then, as shown, it becomes the solid of ① when I put the solids of A, B, ⑤ together.

I can express the volume in the next expression. It is A + B + C $\times \frac{1}{3}$.

Then, I substitute each solid volume for this expression.

And it becomes the next expression. It is 2 1 6 0^{tsubo} + 2 3 4 0^{tsubo} + 2 5 3 5^{tsubo} $\times \frac{1}{3}$.

It becomes 5345^{tsubo} when I calculate.

⑦ A = 1^{syaku 2sun} \times 1^{syaku 2sun} \times 2^{syaku 5sun}

B = 1^{syaku 2sun} \times 1^{syaku 3sun} \times 2^{syaku 5sun}

C = 1^{syaku 3sun} \times 1^{syaku 3sun} \times 2^{syaku 5sun} $\times \frac{1}{3}$

I substitute these expression for an expression of ⑥

Besides that expression can be transform as follows.

(1^{syaku 2sun} \times 1^{syaku 2sun} + 1^{syaku 2sun} \times 1^{syaku 3sun} + 1^{syaku 3sun} \times 1^{syaku 3sun} $\times \frac{1}{3}$) \times 1^{shaku 5sun}

1^{shaku 5sun} = height

1^{shaku 2sun} = length of the upper base

2^{shaku 5sun} = length of the lower base

1^{shaku 3sun} = lower-upper

So, next expression is led.

(upper \times upper + upper \times remainder + remainder \times remainder $\times \frac{1}{3}$) \times height

I do it as follows here.

upper = a lower = b height = h

{ a \times a + a \times (b - a) + (b - a) \times (b - a) $\times \frac{1}{3}$ } \times h

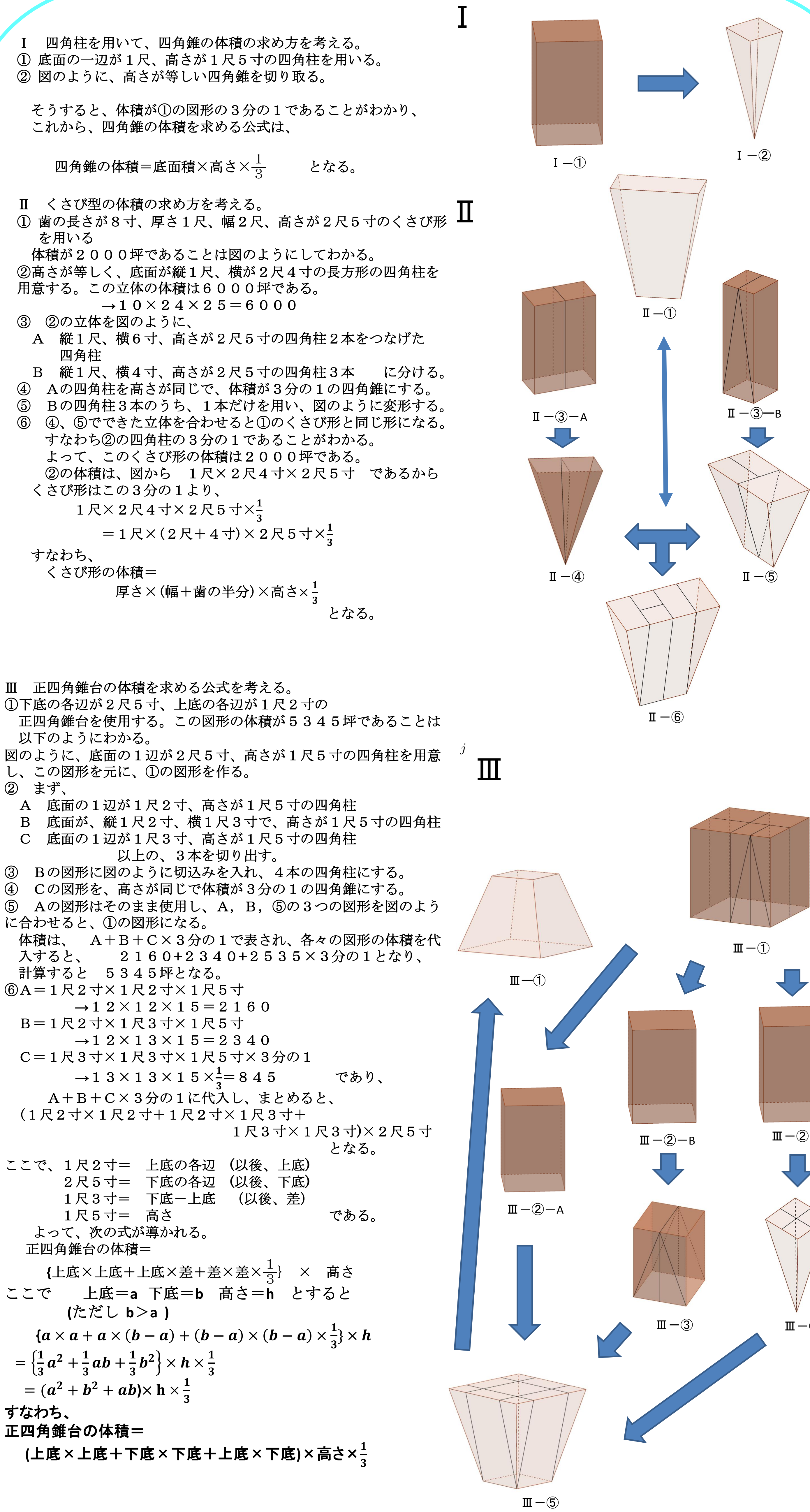
= (a² + b² + ab) \times h $\times \frac{1}{3}$

Volume of the frustum that base is square =

(upper \times upper + lower \times lower + upper \times lower) \times height $\times \frac{1}{3}$

係 海老澤 國定

数学的内容 Mathematical contents



係 河合 國定

まとめ・今後の課題・感想 Summary・Issue In The Future・Impression

まとめ

今回の図形は全て四角錐の考え方を応用しており、どの図形も3分の1をかけている。

All this figures apply the way of the quadrangle pyramid and every figures multiplied by one-third.

今後の課題

文章量が膨大になってしまい、数学的内容、英訳ともに大変な時間がかかってしまった。
わかりやすく要点をまとめることが必要だと感じた。

The quantity of the sentence became enormous.
It took long time to make mathematics translation and English translation.
We felt that it was necessary to compile the main point clearly.

感想

今回調べた、くさび形も正四角錐台も当時の人は、三角錐の考え方を応用して、公式を考えだしたことがわかった。当時の人たちの柔軟な発想力はすごいと思った。
途中さまざまな困難があったが、協力して乗り越えることができ、班全員で一丸となって今回の活動に取り組めたと思う。

We understand that person of those days applied the way of thinking of the tetrahedron to the figure which we checked this time to make formulas.
We thought that the flexible idea power of person of those days was fantastic.
There was various difficulty on the way, but we cooperated and was able to get over it.
We think that we was able to work on this activity with all the groups in solidarity.

班長 河合

算法勿憚改
Sanpou Hutsudankai

延宝元年
A. D. 1673

著者：村瀬 義益
Author：Yoshimasu murase

